Compactness of the Linearized Boltzmann Operator for a Polyatomic Single Gas Model

Abstract

We consider the Boltzmann equation that models a polyatomic gas by taking into account the continuous microscopic internal energy I. In particular, we consider the kinetic system proposed by Bourgat, Desvillettes, Le Tallec, and Perthame (1994), which is based on the procedure of Borgnakke and Larsen (1975). Under some convenient assumptions on the collision cross-section $\mathcal{B}$, we prove that the linearized Boltzmann operator $\mathcal{L}$ is a Fredholm operator. For this, we write $\mathcal{L}$ as $\mathcal{L} = \mathcal{K} - \nu.I$, and we prove that $\mathcal{K}$ is compact. The compactness is achieved as a result of $\mathcal{K}$ being a Hilbert-Schmidt integral operator.