Booklet of abstracts

Partial Differential Equations and Applications

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Hypercontractivity, supercontractivity, ultraboundedness and stability for a class of nonlinear evolution operators

Davide Addona
Università degli Studi di Parma

In my talk I present new results about the improvement in summability of a nonlinear evolution operator in L^p-spaces with respect to tight evolution systems of measures. In particular, I begin with the study the Cauchy problem associated to a family of nonautonomous semilinear equations in the space of bounded and continuous functions over $\mathbb{R}^d$ and in L^p-spaces with respect to the systems of measures quoted above. Here, the linear part of the equation is a nonautonomous second-order elliptic operator with unbounded coefficients defined in $I \times \mathbb{R}^d$, (I being a right-halfline). To the above Cauchy problem we associate a nonlinear evolution operator and we prove that, under suitable assumptions, this evolution operator satisfies hypercontractivity, supercontractivity and ultraboundedness properties. Moreover, as a byproduct we deduce the stability of the null solution to the Cauchy problem.
This is a joint work with L. Angiuli and L. Lorenzi.

On the optimization of traffic flow at a junction

Fabio Ancona
Università di Padova

We consider a traffic flow model at a junction where the dynamics follows the classical conservation law formulation introduced by Lighthill, Whitham and Richards. The usual approach to achieve uniqueness of solutions is based on a pointwise maximization criteria involving the solutions of boundary Riemann problems. We shall address a general optimization problem where the boundary data and the distributional parameters at the junction are regarded as controls. The goal is to select, on a given time interval $[0, T]$, (possibly non unique) solutions which maximize a suitable functional of the flux traces of the incoming edges (as maps over the whole interval $[0, T]$), among all entropy admissible solutions that preserve the conservation of cars through the junction and satisfy some distributional rules.
This is a joint work with A. Cesaroni, G. M. Coclite and M. Garavello.

An overview on linear parabolic systems with unbounded coefficients

Luciana Angiuli
Dipartimento di Matematica e Fisica, Università del Salento, Lecce

In this talk we deal with recent advances in the theory of second order parabolic systems with unbounded coefficients.

Let $\boldsymbol{\mathcal{A}}$ be a second order, uniformly elliptic operator defined on smooth functions $\boldsymbol{\zeta} : \mathbb{R}^d \to \mathbb{R}^m$, $(m \geq 2)$, by

$$(\boldsymbol{\mathcal{A}}\boldsymbol{\zeta})(x) = \sum_{i,j=1}^{d} q_{ij}(x) D_{ij}\boldsymbol{\zeta}(x) + \sum_{i=1}^{d} B_i(x) D_i \boldsymbol{\zeta}(x) + C(x)\boldsymbol{\zeta}(x),$$

where $q_{ij} : \mathbb{R}^d \to \mathbb{R}$ and $B_i, C : \mathbb{R}^d \to \mathbb{R}^{2m}$ for any $i, j = 1, \ldots, d$. Under mild assumptions on the coefficients q_{ij}, B_i and C, it is possible to associate a semigroup $(\mathbf{T}(t))_{t\geq 0}$ to the first-order coupled operator $\boldsymbol{\mathcal{A}}$ in $C_b(\mathbb{R}^d; \mathbb{R}^m)$ (the space of bounded and continuous functions $\mathbf{f} : \mathbb{R}^d \to \mathbb{R}^m$), i.e. for any $\mathbf{f} \in C_b(\mathbb{R}^d; \mathbb{R}^m)$ and $t > 0$, $\mathbf{T}(t)\mathbf{f}$ is the value at t of the unique bounded classical solution of the Cauchy problem $D_t\mathbf{u} = \boldsymbol{\mathcal{A}}\mathbf{u}$ in $(0, +\infty) \times \mathbb{R}^d$ with $\mathbf{u}(0, \cdot) = \mathbf{f}$ in $\mathbb{R}^d$. Some continuity and representation properties of $(\mathbf{T}(t))_{t\geq 0}$ together with some weighted uniform gradient estimates and a sufficient condition for the semigroup to be compact in $C_b(\mathbb{R}^d; \mathbb{R}^m)$ are proved in [2] .

The case of L^p initial data is more difficult. Even in the scalar case, the Cauchy problem associated to $\mathcal{A} = \sum_{i,j=1}^{d} q_{ij} D_{ij} + b_i D_i$ $(q_{ij}, b_i : \mathbb{R}^d \to \mathbb{R})$ may be not well posed in $L^p(\mathbb{R}^d, dx)$ if the coefficients of $\mathcal{A}$ are unbounded, unless they satisfy very restrictive growth assumptions. This is also the situation in the vector-valued case (see [3]). The only way to work in L^p spaces is to replace the Lebesgue measure dx by another measure. The best situation in the scalar case is when there exists an invariant measure μ, namely a Borel probability measure such that

$$\int_{\mathbb{R}^d} T(t) f \, d\mu = \int_{\mathbb{R}^d} f \, d\mu, \quad t > 0, \ f \in C_b(\mathbb{R}^d),$$

where $T(t)$ is the semigroup associated to $\mathcal{A}$ in $C_b(\mathbb{R}^d)$.

Following the ideas of [1], we provide a consistent definition of invariant measures for $(\mathbf{T}(t))_{t\geq 0}$ in $C_b(\mathbb{R}^d; \mathbb{R}^m)$ in the weakly coupled case (i.e. $B_i = b_i I_m$ for some $b_i : \mathbb{R}^d \to \mathbb{R}$ and any $i = 1, \ldots, m$), providing sufficient conditions for the existence of such measures. Moreover we show that, as in the scalar case, the vector-valued semigroup $(\mathbf{T}(t))_{t\geq 0}$ enjoys good properties in the L^p-spaces related to these measures which are also related to the asymptotic behavior of $\mathbf{T}(t)$ as $t \to +\infty$.

These results are obtained in collaboration with D.Addona, L. Lorenzi, D. Pallara and G. Tessitore.

References

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The heat semigroup in $BV(\Omega)$

Viorel Barbu
Romanian Academy

The heat semigroup $\exp(-t\Delta)$ is a continuous semigroup of contractions in the space $BV(\Omega)$ if Ω is convex.

Some results on a linearized model for propagation of ultrasound waves

Francesca Bucci
Università degli Studi di Firenze

In this talk we will deal with a third order (in time) Partial Differential Equation (PDE), which is a linearization of a PDE model for ultrasound propagation known as Jordan-Moore-Gibson-Thompson equation, and is referred to in the recent literature as the Moore-Gibson-Thompson equation (MGT). Semigroup well-posedness, spectral analysis and exponential stability of solutions to the MGT equation – corresponding to certain values of the parameters which occurr in the equation –, complemented with Dirichlet or Neumann boundary conditions, have been obtained by Kaltenbacher, Lasiecka and Marchand in 2011 and by Marchand, McDevitt and Triggiani in 2012. A major focus of attention during the talk will be on an optimal control problem associated with the equation, which is brought about by natural choices of the cost functional (as one seeks to specifically minimize the acoustic pressure), and of the boundary conditions, that are *absorbing* ones on a subpart of the boundary. We will report recently obtained results about its solvability, in spite of the unfavourable combination of a non-analytic semigroup describing the free dynamics, with the unusual pattern displayed by the controlled dynamics.

Possible developments or implications for the study of optimal control problems associated with coupled systems of hyperbolic/parabolic PDE, arising in entirely different physical contexts, will be discussed. Finally, if time permits, a taste will be given of how the intrinsic relation with *viscoelastic* equations, already pointed out, e.g., by Dell'Oro and Pata in 2016, can be exploited in order to derive optimal regularity results for the MGT equation.

The talk is based on ongoing joint work with Irena Lasiecka (University of Memphis, USA) for the former part and with Luciano Pandolfi (Politecnico di Torino) for the latter.

Invariance for quasi-dissipative systems in Banach spaces

Piermarco Cannarsa
University of Rome Tor Vergata

In a separable Banach space, we study the invariance of a closed set K under the action of the evolution equation associated with a maximal dissipative linear operator perturbed by a quasi-dissipative term. Using the distance to the closed set, we give a general necessary and sufficient condition for the invariance of K. Then, we apply our result to several examples of partial differential equations.
This is a joint work with G. Da Prato and H. Frankowska.

Viscoelasticity with a singularly oscillating external force

Monica Conti
Dipartimento di Matematica - Politecnico di Milano

We consider the nonautonomous viscoelastic equation with singularly oscillating external force

$$\partial_{tt}u - \kappa(0)\Delta u - \int_0^\infty \kappa'(s)\Delta u(t-s)\d s + f(u) = g_0(t) + \varepsilon^{-\rho} g_1(t/\varepsilon),$$

where $\rho > 0$ is given and $\varepsilon \in (0, 1]$. Under suitable assumptions on the nonlinearity and on the external force, the related solution processes $S_\varepsilon(t, \tau)$ acting on the natural weak energy space $\mathcal{H}$ are shown to possess uniform attractors $\mathcal{A}^\varepsilon$. We show that the family $\mathcal{A}^\varepsilon$ is bounded in $\mathcal{H}$ uniformly with respect to ε, and we establish the convergence of the attractors $\mathcal{A}^\varepsilon$ to the attractor $\mathcal{A}^0$ of the averaged equation as $\varepsilon \to 0$.
This is a joint work with Prof. V.V. Chepyzhov and Prof. V. Pata.

Elliptic operators with infinitely many variables

Giuseppe Da Prato
Scuola Normale Superiore, Pisa

We shall consider second order elliptic equations in a separable infinite dimensional Hilbert space H as the following one

$$\lambda\varphi - \frac{1}{2}\sum_{h=1}^{\infty} c_h D_h^2\varphi + \sum_{h=1}^{\infty}(\alpha_h x_h + D_h U(x)\, D_h\varphi) = f, \tag{1}$$

where (e_h) is an orthonormal basis in H, $x_h = \langle x, e_h\rangle$, (α_h), (c_h) are positive numbers and $U : H \to \mathbb{R}$ is convex lower semicontinuous. Moreover, f and $\lambda > 0$ are given whereas $\varphi : H \to \mathbb{R}$ is the unknown.

In the first part of the talk we shall present some results of existence, uniqueness and regularity for equation (1) in a space $L^2(H, \nu)$ where ν is an invariant measure of the problem, obtained in a collaboration with A. Lunardi, [3].

The second part will be devoted to Dirichlet and Neumann problems in bounded open subsets $\mathcal{O}$ of H from different papers in collaboration with V. Barbu, A. Lunardi and L. Tubaro, [1], [3], [4].

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Differential equations with separation of variables in the right hand part

Yuli Eidelman
Tel-Aviv University

We consider differential equations in the Banach spaces given in the form

$$v'(t) = Av + \varphi(t)p,$$

where A is an unbounded operator, p is an element of the Banach space and $\varphi(t)$ is a scalar function. We present results on the Cauchy problem and different inverse (identification) problems for such equations. Some of results are extended on the degenerate equations

$$(Mv)'(t) = Av + \varphi(t)p$$

with a singular operator M.
This is a joint work with I.Tikhonov and A.Favini.

On the domain of elliptic operators in subsets of Wiener spaces endowed with a weighted Gaussian measure

Simone Ferrari
Università del Salento, Lecce, Italy.

Let X be a separable Banach space endowed with a non-degenerate centered Gaussian measure μ. The associated Cameron–Martin space is denoted by H. Consider two sufficiently regular convex functions $U : X \to \mathbb{R}$ and $G : X \to \mathbb{R}$. We let $\nu = e^{-U}\mu$ and $\Omega = G^{-1}(-\infty, 0]$. In this talk I will study some properties of the domain of the the self-adjoint operator associated with the quadratic form

$$(\psi, \varphi) \mapsto \int_\Omega \langle \nabla_H \psi, \nabla_H \varphi \rangle_H d\nu \qquad \psi, \varphi \in W^{1,2}(\Omega, \nu). \tag{1}$$

In particular I will show a complete characterization of the Ornstein–Uhlenbeck operator on half-spaces, namely when $U \equiv 0$ and G is an affine function, then the domain of the operator defined via (1) is

$$\{u \in W^{2,2}(\Omega, \mu) \,|\, \langle \nabla_H u(x), \nabla_H G(x) \rangle_H = 0 \text{ for } \rho\text{-a.e. } x \in G^{-1}(0)\},$$

where ρ is the Feyel–de La Pradelle Hausdorff–Gauss surface measure.
This is a joint work with D. Addona and G. Cappa (Università degli studi di Parma, Parma, Italy) .

Multiplicative controllability of nonlinear parabolic equations

Giuseppe Floridia
University of Naples Federico II, Italy

In this talk we discuss the global approximate controllability of the following semilinear reaction-diffusion equations governed via the coefficient of the reaction term

$$\begin{cases} u_t \;=\; u_{xx} \;+\; v(x,t)u \;+\; f(u) & \text{in} \quad Q_T = (0,1) \times (0,T)\,, \quad T > 0, \\ u(0,t) = u(1,t) = 0, & \qquad t \in (0,T), \\ u \;|_{t=0} = u_0 \in H^1_0(0,1). \end{cases} \tag{2}$$

Here $v \in L^\infty(Q_T)$ is a control function (bilinear or multiplicative control), which affects the reaction rate of the process described by (2). The nonlinear term $f : \mathbb{R} \to \mathbb{R}$ is assumed to be a Lipschitz function satisfying $f(0) = 0$.
We extend some nonnegative controllability results by bilinear controls assuming that both the initial and target states admit no more than finitely many changes of sign. In particular, in [1] we obtain the following result.

Theorem 1 *Let $u_0 \in H^1_0(0,1)$. Assume that u_0 has finitely many points of sign change. Consider any target state $u^* \in H^1_0(0,1)$ which has exactly as many points of sign change in the same order as u_0. Then, for any $\eta > 0$, there exists $T = T(\eta, u_0, u^*) > 0$ and a multiplicative control $v = v(\eta, u_0, u^*) \in L^\infty(Q_T)$ such that the respective solution u to* (2) *satisfies*

$$\|u(\cdot,T) - u^*\|_{L^2(0,1)} \;\leq\; \eta.$$

The results contained in [1] permit to approach the multidimensional case with radial symmetry. Our interest in reaction-diffusion problems is partly motivated by the study of mathematical models for tumor growth.
This is a joint work with P. Cannarsa and A.Y. Khapalov.

References

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The Cox-Ingersoll-Ross semigroup for growing initial data

Jerry Goldstein
University of Memphis

The Cox-Ingersoll-Ross (CIR) equation (1985) for bonds is one of the two main deterministic PDEs of mathematical finance, the other being the Black-Merton-Scholes (BMS) equation (1973) for stock options. The semigroup governing BMS is easy to construct and study. But the generator for the CIR semigroup is a sum of four noncommuting generators, and its study presents formidable technical difficulties. The first proof of the strong continuity of the CIR semigroup was finally given in 2016 on the space $C_0[0,\infty)$ by Gisele Goldstein, Jerry Goldstein, Rosa Maria Mininni and Silvia Romanelli [GGMR]. But because of this specialized context, initial data which does not vanish at infinity must be excluded. This excludes the main initial condition used in the original CIR paper. What was needed was an extension of our result to a Banach space which allows functions which can grow at infinity. This talk will present the new results of GGMR which solves this problem.

Reconstruction of a convolution kernel depending also on one space variable in a hyperbolic mixed problem

Davide Guidetti
Dipartimento di Matematica, Università di Bologna

We consider the mixed hyperbolic problem

$$\begin{cases} D_t^2 u(t,x,y) + Au(t,x,y) = \displaystyle\int_0^t k(t-s,x)[Au(s,x,y) + \nabla_y \cdot r(x,y)]ds + f(t,x,y), \\ (t,x,y) \in (0,T) \times (0,\pi) \times \Omega, \\ B_{00}(D_x)u(t,0,y) + B_{01}(D_x)u(t,\pi,y) = B_{10}(D_x)u(t,0,y) + B_{11}(D_x)u(t,\pi,y) = 0, \\ (t,y) \in (0,T) \times \Omega, \\ u(t,x,y') = 0, \quad (t,x,y') \in (0,T) \times (0,\pi) \times \partial\Omega, \\ u(0,x,y) = u_0(x,y), \quad (x,y) \in (0,\pi) \times \Omega, \\ D_t u(0,x,y) = u_1(x,y), \quad (x,y) \in (0,\pi) \times \Omega, \end{cases} \tag{1}$$

in the cylindrical domain $(0,T) \times (0,\pi) \times \Omega$, with Ω open, bounded subset of $\mathbb{R}^n$, with suitably regular boundary $\partial\Omega$. A is a strongly elliptic operator and r is a smooth vector field in $(0,\pi) \times \Omega$. $B_{ij}(D_x)$ $(0 \le i,j \le 1)$ are appropriate differential operators in $(0,T) \times \{j\pi\} \times \Omega$ and a Dirichlet boundary condition is prescribed in $(0,T) \times (0,\pi) \times \partial\Omega$, together with the classical initial time conditions. Given the convolution kernel k, (1) is an integrodifferential hyperbolic system.

We suppose that k is unknown (together with u). We want to reconstruct u and k from the supplementary condition

$$\begin{cases} \displaystyle\int_{\partial\Omega} \left\{ \frac{\partial u}{\partial \nu_A}(t,x,y) + \int_0^t k(t-s,x) \left[\frac{\partial u}{\partial \nu_A}(s,x,y) + r(x,y) \cdot \nu(y) \right] ds \right\} d\sigma(y) = h(t,x), \\ 0 \le t \le T, 0 < x < \pi. \end{cases} \tag{2}$$

A result of existence and uniqueness of a solution (u,k) is given. This should be new, if we consider kernels which do not depend only on t.

On Global Strong Well-Posedness of the Primitive Equations

Matthias Hieber
TU Darmstadt, Germany

In this talk we discuss the primitive equations of the ocean, which are considered to be a fundamental model for many geophysical flows. We develop a framework based on the hydrostatic Stokes operator in the L^p-setting, which allows to deduce global well-posedness results for arbitrary large data belonging to certain classes of function spaces. In addition, we discuss the long-time behaviour of strong solutions.
This is a joint work with Y. Giga, M. Gries, A. Hussein and T. Kashiwabara.

Mean Periodic Solutions of a Inhomogeneous Heat Equation with Random Coefficients

Galina Kurina
Voronezh State University

On the space $\mathbb{R}_+ \times \mathbb{R}$ consider the Cauchy problem

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + \varepsilon(t)y + f(t,x), \tag{1}$$

$$y(0,x) = y_0(x), \tag{2}$$

where $\varepsilon(t)$ and $f(t,x)$ are independent random processes given by the characteristic functionals $\varphi_\varepsilon(u)$ and $\varphi_f(v)$ $(u, v \in L_1(\mathbb{C}))$.

A solution of problem (4), (5) is a random process. It is said to be mean periodic with respect to t if its expectation is a periodic function with respect to t.

We study the mean periodicity for two cases.

1) The random process ε is Gaussian with the characteristic functional of the form

$$\varphi_\varepsilon(u) = \exp(i\int_{\mathbb{R}} M(\varepsilon(s))u(s)ds - \frac{1}{2}\int_{\mathbb{R}}\int_{\mathbb{R}} b(s_1,s_2)u(s_1)u(s_2)ds_1ds_2), \tag{3}$$

where $M(\varepsilon(s))$ is the expectation, and $b(s_1,s_2) = M(\varepsilon(s_1)\varepsilon(s_2)) - M(\varepsilon(s_1))M(\varepsilon(s_2))$ is the covariation function of the random process ε.

2) The random process ε is uniformly distributed with the characteristic functional given by the formula

$$\varphi_\varepsilon(u) = \frac{\sin\int_{\mathbb{R}} a(s)u(s)ds}{\int_{\mathbb{R}} a(s)u(s)ds}\exp(i\int_{\mathbb{R}}\xi(s)u(s)ds), \tag{4}$$

where $a(s) \geq 0$ and $\xi(s) = M(\varepsilon(s))$ are continuous functions.

If $a(s) \equiv 0$, then we assume that $\varphi_\varepsilon(u) = \exp(i\int_{\mathbb{R}}\xi(s)u(s)ds)$.

Consider the following operators: $U_x(t)$ is defined by the relation

$$U_x(t)z(\nu,x) = \frac{1}{2\sqrt{\pi t}}\int_{\mathbb{R}}\exp\Big(-\frac{\tau^2}{4t}\Big)z(\nu,x-\tau)d\tau, \quad t>0,$$

$$U_x(0) = I,$$

and the operator $W(t,s)$ is given by the following way

$$W(t,s)(z(\nu,x)\varphi(u)) = U_x(t)U_x^{-1}(s)z(\nu,x)V(t,s)\varphi(u),$$

where the operator $V(t,s)$ is introduced as

$$V(t,s)\varphi(u) = \varphi(u - i\chi(s,t)).$$

The function $\chi(s,t,\tau) = \chi(s,t)(\tau)$ is defined as follows: $\chi(s,t,\tau)$ is equal to $sign(\tau - s)$ for τ belonging to the segment with endpoints s and t and to zero for other τ.

Theorem 2 *Let $M(f(t,x))$ be ω-periodic with respect to t, and the operator $I - W(\omega,0)$ be invertible. If $\varphi_\varepsilon(u)$ has the form (3), where $M(\varepsilon(s))$ is an ω-periodic function, $b(s_1,s_2)$ is an ω-periodic function of both variables or $\varphi_\varepsilon(u)$ has the form (4), where $a(s)$, $\xi(s)$ are ω-periodic functions, then*

$$M(y(t,x)) = W(t,0)(W(0,\omega) - I)^{-1}\int_t^{t+\omega} U_x^{-1}(s)(M(f(s,x))\varphi_\varepsilon(i\chi(0,s))ds$$

is the ω-periodic with respect to t expectation of a solution of equation (4).

This is a joint work with V. Zadorozhniy.

Rigidity and stability results for Gauss-type mean value formulas

Ermanno Lanconelli
Università di Bologna

We show some rigidity results for weighted mean value formulas for harmonic functions in open subsets of $\mathbb{R}^n$. We also show a stability estimate for the classical Gauss mean value formula.

Surface measures in Hilbert spaces

Alessandra Lunardi
Università di Parma

I will describe recent results on the construction and properties of surface measures in infinite dimensional Hilbert spaces endowed with Borel probability measures. The aim is to prove integration by parts formulae for smooth enough (Sobolev) functions defined in good sets, that involve surface integrals. The current literature deals mainly with Gaussian measures, while we emphasize the non Gaussian case. The general theory is applied to a number of different examples, such as weighted Gaussian measures, non-Gaussian product measures, invariant measures of stochastic PDEs.
This is a joint work with Giuseppe Da Prato.

Evolution equations and optimal control: the state constrained case

Elsa Maria Marchini
Politecnico di Milano

Some new results on optimal control problem under state constraints are discussed. We study differential inclusions of the type

$$\dot{x}(t) \in Ax(t) + F(t, x(t)), \tag{1}$$

with $x(t) \in K$. The setting is quite general, hence our analysis applies to some interesting and delicate frameworks: the operator A is the infinitesimal generator of a strongly continuous semigroup $S(t) : X \to X$, and X is an infinite dimensional separable Banach space.

Applications of our results deal with regularity of the value function associated to optimal control problems, and variational inclusions, together with non degenerate first order necessary optimality conditions. The main technical tools are neighboring feasible trajectories theorems, which allow to approximate trajectories of (1) by trajectories lying in the interior of K. The state constraint K (allowed to be nonsmooth), is required to satisfy an *inward pointing condition*.

Our analysis will be applied to models involving different kind PDEs.
This is a joint work with Hèléne Frankowska and Marco Mazzola.

An inverse problem for image denoising

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We propose a technique for image denoising by recovering an original image from a blurred observed one. This turns out to solve a nonconvex optimal control problem with the state and controller connected by a nonlinear elliptic equation with a possible discontinuous diffusion term. The existence of a control able to denoise an initial blurred image is proved and the optimality conditions are determined by a passing to the limit technique in an appropriate approximating problem, involving the Legendre–Fenchel relations.

Spectral properties of non-selfadjoint extensions of the Calogero Hamiltonian

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We describe all extensions of the Calogero Hamiltonian

$$L = -\frac{d^2}{dr^2} + \frac{b}{r^2} \quad \text{in } L^2(\mathbb{R}_+), \quad b < -\frac{1}{4}$$

having non empty resolvent set and generating an analytic semigroup in $L^2(\mathbb{R}_+)$. Some extensions to the N-dimesnional setting are also given.
This is a joint work with Motohiro Sobajima, Tokyo University of Science, Japan.

The Cahn-Hilliard equation in image inpainting

Alain Miranville
Université de Poitiers, France

Our aim in this talk is to discuss variants of the Cahn-Hilliard equation in view of applications to image inpainting. We will present theoretical results as well as numerical simulations.

Qualitative and Quantitative Analysis for a Nonlinear Reaction-Diffusion Equation with Constant Coefficients

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In the present work we are concerned to treat the existence, uniqueness, regularity and the approximation of solutions to the reaction-diffusion equation with constant coefficients, endowed with a cubic nonlinearity and Neumann boundary conditions, relevant in a wide class of physical phenomena, including *phase separation* and *transition.* Regarding the approximation of solution, the convergence of a new iterative scheme of fractional steps type, is also established. We prove L^∞ stability by maximum principle arguments and derive error estimates using energy methods for the implicit Euler and, tree implicit-explicit approaches: a linearized scheme and two different schemes of fractional steps type. Some numerical experiments was done in order to validates the theoretical results and to compare the accuracy of the numerical methods above mentioned.

Fractional equations with logistic–type nonlinearities

Dimitri Mugnai
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We consider existence and multiplicity results for a semilinear problem driven by the square root of the negative Laplacian in presence of a nonlinear term which is the difference of two powers. In the case of convex–concave powers, a precise description of the problem at the threshold value of a given parameter is established through variational methods and truncation techniques.
This is a joint work with Giulia Carboni.

Blagoveshchenskiĭ equation and Identification of a space varying coefficient of a linear viscoelastic string

Luciano Pandolfi
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Politecnico di Torino, Torino - Italy

In the equation

$$w_t = \int_0^t N(t-s)\mathcal{L}w(s)\mathrm{d}\,s\,, \qquad \mathcal{L} = \Delta + q(x)\,, \quad x \in (0,L)\,, \tag{1}$$

$w = w(x,t)$ represents the displacement of a viscoelastic string at time t and position x and $N(t)$ is the *relaxation kernel.* We assume zero initial conditions, $w(x,0) = 0$, and we assume that the system is excited by applying a deformation on the boundary:

$$w(0,t) = f(t)\,, \qquad w(L,t) = 0\,. \tag{2}$$

Let $w^f(x,t)$ be the solution of (1)-(2) (null initial conditions).

A problem which is much studied in the case of the wave equation, i.e. when $N(t) \equiv 1$, is the identification of the *unknown coefficient* $q(x)$. Several different procedures exist and we concentrate on a procedure first proposed by Belishev, and which is based on the controllability property of the wave equation (with the boundary conditions (2)), a property which is shared by Eq. (1) (when $N(t)$ is smooth with $N(0) > 0$).

The identification of the coefficient $q(x)$ require an additional information, and it is assumed that we know the *reaction operator* (the operator is unbounded, with domain $H^1(0,T)$)

$$f \mapsto w_x^f(0,t) = Rf\,, \qquad L^2(0,T) \mapsto L^2(0,T)\,. \tag{3}$$

The controllability property is as follows: for every $\tau \in (0,L]$

$$\{w^f(\cdot,\tau)\,, \quad f \in L^2(0,\tau)\} = L^2(0,\tau)\,.$$

Let us consider the *controllability operator* i.e. the quadratic form

$$H_\tau(f,g) = \int_0^\tau w^f(x,s)w^g(x,s)\mathrm{d}\,s\,.$$

In the context of inverse problems this operator is called the *connecting operator.*

The properties of the wave equation and controllability imply that this operator is bounded and surjective. In fact, if $\tau \in (0,L]$ it is also boundedly invertible.

Belishev proposed an algorithm to reconstruct $q(x)$ from the family of operators H_τ which can be extended without difficulty from the wave equation (i.e. the case $N(t) \equiv 1$) to the general case.

So, the problem of the identification of $q(x)$ is reduced to see whether the operators H_τ can be computed. It is possible to see that *in the case of the wave equation, the operators H_τ can be computed from the response operator* via the solution of a simple PDE, first introduced by Blagoveshchenskiĭ. *The important fact is that this equation does not explicitly depend on* $q(x)$ (the effect of the unknown coefficient $q(x)$ is hidden behind the reaction operator, which can be experimentally measured).

We show that an analogous of Blagoveshchenskiĭ equation exists also in the general case $N(t) \neq 1$. This is a PDE equation with "memory" both in the time and space variables and we prove that it is uniquely solvable. The structure of the equation (which is "causal" in the lexographyc order) shows that it can be solved in a recursive way, using a "marching" algorithm in space and time.

Flocking results for the Cucker–Smale model with time delay

Cristina Pignotti
Università di L'Aquila

The Cucker–Smale model has been proposed in 2007 as a model for flocking, namely for phenomena where autonomous agents reach a consensus based on limited environmental information. We will describe a CS–model with time delay. Indeed, it is natural to assume that the information from other agents is received after a time lag or that every agent needs a time to elaborate it. We consider a time dependent delay $\tau(t)$ since we may expect that the size of the delay exhibits some seasonal effects or e.g. it depends on the age of the agents. Under appropriate assumptions, we will give flocking results if the time delay satisfies a suitable upper bound. We will discuss two different approaches to deal with both symmetric or nonsymmetric communication weights between the agents.
This is a joint work with E. Trélat.

Sharp Estimates for Geman-Yor Processes and applications to Arithmetic Average Asian options

Sergio Polidoro
Università degli Studi di Modena e Reggio Emilia (Italy)

We prove the existence and pointwise lower and upper bounds for the fundamental solution of the degenerate second order partial differential equation related to Geman-Yor stochastic processes, that arise in models for option pricing theory in finance.
Lower bounds are obtained by using repeatedly an invariant Harnack inequality and by solving an associated optimal control problem with quadratic cost. Upper bounds are obtained by the fact that the optimal cost satisfies a specific Hamilton-Jacobi-Bellman equation.
This is a joint work with G. Cibelli, and F. Rossi.

Parabolic estimates and Poisson process

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Abstract: We show among other things how knowing Schauder or Sobolev-space estimates for the one-dimensional heat equation allows one to derive their multidimensional analogs for equations with coefficients depending only on time variable with the *same* constants as in the case of the one-dimensional heat equation. The method is quite general and is based on using the Poisson stochastic process. It also applies to equations involving non-local operators. It looks like no other method is available at this time and it is a very challenging problem to find a purely analytic approach to proving such results.
This is a joint work with N.V. Krylov (University of Minnesota)

On the quasi-geostrophic equations on compact surfaces in $\mathbb{R}^3$

Jan Prüss
Martin-Luther Universität Halle-Wittenberg, Germany

We consider an approach via maximal L_p-regularity to the quasi-geostrophic equations on compact surfaces in $\mathbb{R}^3$, like the sphere S^2 or the torus T^2. Based on methods from the theory of parabolic evolution equations, we prove global well-posedness and global exponential stability of solutions starting in the critical spaces, which are well-known for this problem. We also obtain real analyticity of the solutions jointly in space and time if the underlying surface has this property.

L^p-theory for Schrödinger systems

Abdelaziz Rhandi
University of Salerno

In this talk we study for $p \in (1, \infty)$ the L^p-realization of the vector-valued Schrödinger operator $\mathcal{L}u := \mathrm{div}(Q\nabla u) + Vu$. Using a noncommutative version of the Dore–Venni theorem due to Monniaux and Prüss, we prove that the L^p-realization of $\mathcal{L}$, defined on the intersection of the natural domains of the differential and multiplication operators which form $\mathcal{L}$, generates a strongly continuous contraction semigroup on $L^p(\mathbb{R}^d; \mathbb{C}^m)$. We also study additional properties of the semigroup such as extension to L^1, positivity and ultracontractivity and prove that the generator has compact resolvent. We end by giving several examples and counterexamples.

This is a joint work with Markus Kunze, Luca Lorenzi and Abdallah Maichine.

General Wentzell boundary conditions with heat flow on the boundary

Silvia Romanelli
Università degli Studi di Bari Aldo Moro

We will present some recent results obtained with A. Favini, G. R. Goldstein, J. A. Goldstein and E. Obrecht (see [1]), concerning different versions of elliptic operators M_A equipped with general Wentzell boundary conditions (GWBC) of the type

$$M_A u + \beta \partial_\nu^A u + \gamma u - q\beta \Delta_{LB} u = 0, \ \textit{on } \partial\Omega.$$

Here Ω is a domain of $\mathbb{R}^N$ having its nonempty boundary $\partial\Omega$ consisting of sufficiently smooth $(N-1)$ dimensional manifolds, ν is the unit outer normal on $\partial\Omega$, $\partial_\nu^A u$ is the conormal derivative of u with respect to a suitable $N \times N$ real Hermitian matrix A associated with M_A. Moreover β, γ are suitable bounded regular functions on $\partial\Omega$, $\beta > 0$, $q \in [0, \infty)$ and Δ_{LB} is the Laplace Beltrami operator. The main focus will be on sufficient conditions for the generation of analytic semigroups in spaces of L^p type, $1 < p < \infty$. Note that, according to the physical interpretation given by G. R. Goldstein in [2], if $q \neq 0$, the presence of the term $\Delta_{LB} u$ corresponds to the existence of a heat flow on the boundary. Extensions and related results will be also discussed.

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Stability for the electromagnetic scattering problem

Luca Rondi
Università di Trieste, Italy

I discuss the scattering problem for time-harmonic electromagnetic waves, due to the presence of scatterers and of inhomogeneities in the medium. I show a stability result for the solution to the corresponding exterior boundary value problem, with respect to variations of the scatterer and of the inhomogeneity, under minimal regularity assumptions for both of them. For example, both obstacles and screen-type scatterers are allowed at the same time.
This is a joint work with Hongyu Liu and Jingni Xiao (Hong Kong Baptist University).

Instantaneous Blowup in $\mathbb{R}^N$ and $\mathbb{H}^N$

Gisèle Ruiz Goldstein
University of Memphis

Consider the heat equation

$$\frac{\partial u}{\partial t} = \Delta u + V(x)u$$

for $x \in \mathbb{R}^N$ with a positive potential $V(x)$. If V is "too singular", then this equation may not have any positive solutions, as was discovered in 1984. We shall discuss the history of the problem as well as later developments, including new results obtained in 2016-17. The Euclidean space $\mathbb{R}^N$ can be replaced by the Heisenberg group $\mathbb{H}^N$ and other Carnot groups, and the heat equation can be relaced by the Ornstein-Uhlenbeck equation and other related equations. Some nonlinear results will be mentioned. Scaling plays a critical role.

Polynomial stability of two coupled strings

Roland Schnaubelt
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We consider two vibrating strings which are coupled at a common boundary point. This coupling is the only source of damping in the system. It has been known that this system is strongly, but not exponentially, stable if the quotient d of the wave speeds of the two springs is irrational. We show that classical solutions polynomially tend to 0 with a uniform rate, where the decay exponent depends on number theoretic properties of the material constant d. The proof is based on a characterization of polynomial stability due to Borichev-Tomilov and Batty-Duyckaerts. To apply it, one has to bound the resolvent of the generator of the corresponding semigroup along the imaginary axis. The main step is an analysis of a complex function characterizing the spectrum, which involves rational approximations of d.

This is joint work with Łukasz Rzepnicki (Torún).

Elliptic Functional Differential Equations and the Kato Square Root Problem

A.L. Skubachevskii
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In 1961, T. Kato has formulated the following problem: "Is it true that a domain of square root of regular accretive operator is equal to a domain of square root of adjoint operator?" J.-L. Lions has obtained sufficient conditions for fulfilment of the Kato conjecture for abstract regular accretive operators. He has also proved that strongly elliptic differential operators with smooth coefficients and the Dirichlet conditions in a bounded domain with smooth boundary satisfy the Kato conjecture. A proof was based on the theorem on smoothness of generalized solutions to elliptic problems, which allows to present a domain of strongly elliptic differential operator in explicit form, and on the interpolation theory. In 1972, A. McIntosh has constructed a counterexample of abstract regular accretive operator that does not satisfy the Kato conjecture. Therefore, further mathematicians tried to find new classes of operators satisfying the Kato conjecture. For strongly elliptic differential operators with measurable bounded coefficients, a corresponding result was obtained by P. Auscher, S. Hofman, A. McIntosh, and P. Tchamitchian in 2001. The main difficulties were related to an absence of smoothness for generalized solutions. Therefore, it was impossible to present a domain of operator in explicit form.

In 2002, W.Arendt has called my attention to the fact that strongly elliptic differential-difference operators [1] are also satisfying the Kato conjecture. Due to nonlocal nature of these operators, smoothness of generalized solutions to corresponding equations can be violated inside a domain. In this lecture, we shall give a review of results related to the Kato conjecture for strongly elliptic functional-differential operators [2] and formulate new results concerning the Kato conjecture for elliptic differential-difference equations with degeneration. Elliptic differential-difference equations with degeneration have some astonishing properties. For example, smoothness of generalized solutions to corresponding equations do not belong even to the Sobolev space of the first order.

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Elliptic operators with unbounded diffusion, drift and potential terms

Cristian Tacelli
Università di Salerno

We prove that the realization A_p in $L^p(\mathbb{R}^N)$, $1 < p < \infty$, of the elliptic operator $A = (1 + |x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|} \cdot \nabla - c|x|^\beta$ with domain $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N) \,|\, Au \in L^p(\mathbb{R}^N)\}$ generates a strongly continuous analytic semigroup $T(\cdot)$ provided that $\alpha > 2$, $\beta > \alpha - 2$, $b \in \mathbb{R}$ and $c > 0$. This extends the recent results in [1] and [2]. Moreover we show that $T(\cdot)$ is consistent, immediately compact and ultracontractive.
This is a joint work with S.E. Boutiah, F. Gregorio and A. Rhandi.

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Spectral analysis of the subelliptic oblique derivative problem

Kazuaki Taira
University of Tsukuba, Japan

My talk is devoted to a functional analytic approach to the subelliptic oblique derivative problem for the Laplacian with a complex parameter λ. We prove an existence and uniqueness theorem of the homogeneous oblique derivative problem in the framework of L^p Sobolev spaces when $|\lambda|$ tends to ∞. In the proof we make use of Agmon's technique of treating a spectral parameter λ as a second-order elliptic differential operator of an extra variable on the unit circle and relating the old problem to a new one with the additional variable. As an application of the main theorem, we prove generation theorems of analytic semigroups for this subelliptic oblique derivative problem in the L^p topology and in the topology of uniform convergence. Moreover, we solve the long-standing open problem of the asymptotic eigenvalue distribution for the homogeneous oblique derivative problem when $|\lambda|$ tends to ∞. We prove the spectral properties of the closed realization of the Laplacian similar to the elliptic (non-degenerate) case. In the proof we make use of Boutet de Monvel calculus in order to study the resolvents and their adjoints in the framework of L^2 Sobolev spaces.

Controllability of the beam equation

Cristina Urbani
GSSI - Gran Sasso Science Institute

Analysis of a method used by K. Beauchard to prove the controllability of the Schrödinger equation. Application of this procedure to gain the controllability of the beam equation with an easier proof and in a more general space with respect to previous results.
This is a joint work with P. Cannarsa.

Laplace Reaction-Diffusion Equations

Atsushi Yagi
Osaka University, Professor Emeritus

The Łojasiewicz-Simon theory is going to be extended into an infinite-dimensional version. The extended one is then available to various nonlinear partial differential equations which admit an analytic Lyapunov function (see, e.g., [1], [2]).

In this talk, we want to treat the initial-boundary value problem for a nonlinear elliptic parabolic equation of the form

$$\begin{cases} m(x)\dfrac{\partial u}{\partial t} = a\Delta u + m(x)f(u) & \text{in} \quad \Omega \times (0,\infty), \\ u = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in} \quad \Omega, \end{cases} \tag{4}$$

in a three-dimensional bounded domain Ω. Here, $m(x)$ is a given function in Ω such that $0 \le m(x) \in L_\infty(\Omega)$ and $m(x) \not\equiv 0$; $a > 0$ is a diffusion constant; and $u_0(x)$ is an initial function in Ω satisfying $m(x)u_0(x) \ge 0$. The function $f(u)$ is a real analytic function such that $f(u) \le D_0$ and $f'(u) \le D_1$ for $0 \le u < \infty$ with some constants $D_i > 0\,(i = 0, 1)$.

In order to employ the theory of abstract parabolic equations, we formulate (4) as the Cauchy problem for a semilinear equation

$$\begin{cases} \dfrac{du}{dt} + Au \ni f(u), & 0 < t < \infty, \\ u(0) = u_0, \end{cases} \tag{5}$$

in $H_0^1(\Omega)$, where $A = -a[m(x)^{-1}]\Delta$ is a multivalued linear operator of $H_0^1(\Omega)$. As shown by [3] and [4], A generates an analytic semigroup on $H_0^1(\Omega)$. Then, we can solve (5) by using the techniques for abstract parabolic semilinear equations, see [5].

Our goal is to see the global existence of strict solution to (5) and to construct a Lyapunov function. Indeed, it is proved that, for some $\frac{1}{2} < \beta < 1$, $K = \{u \in \mathcal{D}(A^\beta); m(x)u \ge 0\}$ becomes a phase space of the dynamical system generated by (5) and

$$\Psi(u) = \int_\Omega \left[\frac{a}{2}|\nabla u|^2 - m(x)F(u)\right] dx, \qquad u \in \mathcal{D}(A^\beta),$$

where $F(u) = \int_0^u f(v)dv$, is a Lyapunov function on K.

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Lipschitz stability for inverse source problems for hyperbolic-parabolic systems in fluid dynamics

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Hyperbolic-parabolic systems appear often in mathematical physics. Here we discuss such two systems in fluid dynamics.

I. Non-homogeneous Navier-Stokes equations.

$$\begin{cases} \partial_t \rho(x,t) + v(x,t)\cdot\nabla\rho = 0, \\ \rho(x,t)\partial_t v + \rho(v\cdot\nabla)v = \Delta v + \nabla p + R(x,t)f(x), \\ \operatorname{div} v = 0 \quad \text{in } Q := \Omega\times(0,T). \end{cases} \tag{1}$$

II. Compressible viscous fluid in isothemral case.

$$\begin{cases} \partial_t \rho(x,t) + \operatorname{div}(\rho(x,t)v(x,t)) = 0, \\ \rho(x,t)\partial_t v = \mu(x)\Delta v + \lambda(x)\nabla(\operatorname{div} v) - \rho(v\cdot\nabla)v + g(x,t)\nabla\rho + R(x,t)f(x) \quad \text{in } Q. \end{cases} \tag{2}$$

We attach suitable boundary conditions for each (1) and (2).

We consider

Inverse Problem.

Determine $f(x)$, $x \in \Omega$ by suitable data $(\rho, v)|_{\omega\times(0,T)}$ and $(\rho, v)(\cdot, t_0)$.

Here let R be given and satisfy $\det R(\cdot, t_0) \neq 0$ on $\overline{\Omega}$, and let $\omega \subset \Omega$ be a suitable subdomain, $0 < t_0 < T$ be given.

By the conventional method on the basis of Carleman estimates, we can prove at best the Hölder stability for the inverse problems.

Our main result is the global Lipschitz stability, and we show how to establish the Lipschitz stability for other inverse problem of determining Lamé coefficients $\lambda(x), \mu(x)$ for (2) by similar data.

This is a joint work with Oleg Imanuvilov (Colorado State University, USA).

Sobolev Type Equations of Higher Order and Applications

Alyona A. Zamyshlyaeva
South Ural State University, Chelyabinsk, Russia

Of concern are semilinear Sobolev type equations of higher order with the Cauchy initial conditions or the Showalter – Sidorov conditions which are more natural for the Sobolev type equations. Moreover we consider an initial-final problem when some projection of the solution is defined at initial point and the other projection is defined at the final point of time period.

Such equations are investigated in variety of aspects. Our approach of the Cauchy problem study is based on a phase space concept, the essence of which lies in a reduction of singular equation to a regular one defined, however, not on a whole space but on some subset of initial space, containing all initial values. To describe the morphology of the phase space of the initial equation, it may seem that it is sufficient to reduce this equation using the standard procedure to a linear equation of the first order, the phase spaces of which are well studied. However, on that way there arise unexpected difficulties: it turns out that in some cases for the solvability of the problem the Cauchy conditions need to be related. We should emphasize that there is no such a phenomena in the description of phase spaces of Sobolev type equations of the first order and classical equations.

We are also interested in the optimal control problem for such equations. Abstract results are applied to the variety of mathematical models, for example based on: the De Gennes equation of the acoustic waves in a smectic; the Boussinesq – L'ove equation which describes shallow water wave propagation if defined in a domain or the vibration processes in a construction made of thin elastic rods if defined on a geometrical graph; equation of ion-acoustic waves in plasma in external magnetic field and other.

This is a joint work with E.Bychkov and O.Tsyplenkova.